Substituting in this equation, the values of x, y, and z, and reducing, we obtain

$$r(bc+ac+ab-a^2-b^2-c^2)=bc+ac+ab-a^2-b^2-c^2.$$

From this equation we find that $r=1$, unless

$$\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix}=0\ldots.(2).$$

But (2) is the condition that the triangle a, b, c is equilateral. Therefore, either $r=1$, or else both triangles are equilateral.

Also solved by *G. B. M. ZERR*, *J. W. YOUNG*, and *FRANK A. GRIFFIN*.

131. Proposed by J. W. YOUNG, Fellow and Assistant in Mathematics, Ohio State University, Columbus, O.

Prove that $\lambda+\mu\omega+\nu\omega^2$, where λ, μ, ν are integers whose sum is ±1, represents the points of a quilt formed by regular hexagons. ω=primitive cube root of unity. [From Harkness and Morley's *Introduction to Theory of Functions.*]

Solution by the PROPOSER.

$$\omega=-\tfrac{1}{2}+i(\tfrac{1}{2}\sqrt{3}),\ \omega=-\tfrac{1}{2}-i(\tfrac{1}{2}\sqrt{3})\ (i^2=-1).$$

Then $\lambda+\mu\omega+\nu\omega^2=\lambda-\frac{1}{2}\mu-\frac{1}{2}\nu+i(\mu-\nu)\sin\frac{1}{3}\pi$. Taking rectangular coördinates this quantity represents the points (x, y), when

$$\left.\begin{aligned} x&=\lambda-\tfrac{1}{2}\mu-\tfrac{1}{2}\nu \\ y&=(\mu-\nu)\sin\tfrac{1}{3}\pi \end{aligned}\right\}\begin{aligned}&\lambda+\mu+\nu=\pm1\\ &(\lambda,\ \mu,\ \nu \text{ are integers})\end{aligned}$$

$y=n\sin\frac{1}{3}\pi$, when $n=(\mu-\nu)$=any integer.
Then $\mu=n+\nu$.
Substituting in x and in $\lambda+\mu+\nu=\pm1$, we obtain

$$\left.\begin{aligned} 2\lambda-2\nu-n&=2x \\ \lambda+2\nu+n&=\pm1 \end{aligned}\right\}\ldots.(1).$$

$\therefore x=\frac{1}{3}(3\lambda\pm1)$.

The points required are, then, those whose coördinates are $[\frac{1}{3}(3\lambda\pm1), n\sin\frac{1}{3}\pi]$, where λ, n are any integers with the one restriction that when n is *odd*, λ is even, and *vice versa*. This restriction is evident, since (1) shows that $\lambda+n$ must be *odd*.

We have then following values of x and y :

A. For $n=0, 2, 4, 6$, etc.

$\lambda=$	1	3	5	7	9	etc.
$x=$	1 2	4 5	7 8	10 11	13 14	

For $n=1, 3, 5, 7$, etc.

$\lambda=$	0	2	4	6	8	
$x=$	$-\frac{1}{2}$ $\frac{1}{2}$	$\frac{5}{2}$ $\frac{7}{2}$	$\frac{11}{2}$ $\frac{13}{2}$	$\frac{17}{2}$ $\frac{19}{2}$	$\frac{23}{2}$ $\frac{25}{2}$	

$y=n\sin\frac{1}{2}\pi$.

Similarly for negative values of n, λ.

MECHANICS.

96. Proposed by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Two particles, subject to their mutual attraction and that of a fixed center, move in a plane containing the center. Find the motion under the law of the inverse square.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Take the center of force as origin. Let m, m_1, m_2 be the masses of the center of force and particles, respectively. r, r_1, ρ the distances of the particles from the center of force and from each other, respectively. (x, y), (x', y') the coördinates of the particles. The differential equations of motion of the two particles relative to the center of force are

$$\left.\begin{aligned}\frac{d^2x}{dt^2}&=-\frac{m+m_1}{r^3}x+\frac{x'-x}{\rho^3}m_2-\frac{m_2x'}{r_1^3}\\ \frac{d^2y}{dt^2}&=-\frac{m+m_1}{r^3}y+\frac{y'-y}{\rho^3}m_2-\frac{m_2y'}{r_1^3}\end{aligned}\right\}\ldots.(1).$$

$$\left.\begin{aligned}\frac{d^2x'}{dt^2}&=-\frac{m+m_2}{r_1^3}x'+\frac{x-x'}{\rho^3}m_1-\frac{m_1x}{r^3}\\ \frac{d^2y}{dt^2}&=-\frac{m+m_2}{r_1^3}y'+\frac{y-y'}{\rho^3}m_1-\frac{m_1y}{r^3}\end{aligned}\right\}\ldots(2).$$

Where $\rho=\sqrt{[(x'-x)^2+(y'-y)^2]}$.

Multiply (1) by $2m_1\dfrac{dx}{dt}-2m_1\dfrac{m_1\dfrac{dx}{dt}+m_2\dfrac{dx'}{dt}}{m+m_1+m_2}$.

$$2m_1\frac{dy}{dt}-2m_1\frac{m_1\dfrac{dy}{dt}+m_2\dfrac{dy'}{dt}}{m+m_1+m_2}.$$